On the determination of $\mathcal{CP}$-even and $\mathcal{CP}$-odd components of a mixed $\mathcal{CP}$ Higgs boson at e^+e^- linear colliders

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We present a method to investigate the $\mathcal{CP}$ quantum numbers of the Higgs boson in the process $e^+e^- \to Z\phi$ at a future e^+e^- linear collider (LC), where ϕ, a generic Higgs boson, is a mixture of $\mathcal{CP}$-even and $\mathcal{CP}$-odd states. The procedure consists of a comparison of the data with predictions obtained from Monte Carlo simulations corresponding to the productions of scalar and pseudoescalar Higgs and the interference term which constitutes a distinctive signal of $\mathcal{CP}$ violation. We present estimates of the sensitivity of the method from Monte Carlo studies using hypothetical data samples with a full LC detector simulation taking into account the background signals.

INTRODUCTION

The future linear e^+ e^- collider TESLA is planned to work with a maximum center-of-mass energy of 500 GeV, extendable to 800 GeV without modifying the original design [1]. It will have a luminosity of 3.4×10^{34} cm^{-2} s^{-1}, a thousand times greater than the LEP at CERN, and so it will be well suited for a discovery of a light Higgs boson. Even if the Higgs is discovered before at Tevatron (Fermilab) [1] or at the future LHC (CERN, Geneva), $e+e-$ colliders are the ideal machines to investigate the Higgs sector in the intermediate mass range since all major decay modes can be explored, with the Higgs particle produced through several mechanisms [3]. For a light or intermediate mass Higgs boson, the Higgsstrahlung process $e^+e^- \to Z\Phi$, where Φ denotes a generic Higgs boson, is expected to be the most promising process to study its properties and interactions and to search for deviations from the Standard Model (SM) predictions(see [4] and references therein). A comprehensive review of the Higgs boson properties has been given in ref. [5]. The theory of the Higgs bosons, with emphasis on the Higgs scalars of the SM and its non-supersymmetric and supersymmetric extensions has been recently presented in ref.[6]. The spin, parity and charge conjugation quantum numbers, J^{PC}, of the Higgs boson can potentially be determined independently of the model. It has been shown that measurements of the threshold dependence of the Higgsstrahlung cross-section constrains the possibles values J^{PC} of the state [7]. In the minimal Standard Model the Higgs mechanism requires only one Higgs doublet to generate masses for fermions and gauge bosons [8]. It leads to the appearence of a neutral $\mathcal{CP}$-even Higgs (H). In the two-doublet Higgs model (2DHM) or the supersymmetric extension of the SM [9], neglecting $\mathcal{CP}$ violation, there are two $\mathcal{CP}$-even states (h, H) and one $\mathcal{CP}$-odd state (A), plus a pair of charged Higgs bosons

($H^\pm$). In a general 2DHM the three neutral Higgs bosons could correspond to arbitrary mixtures of $\mathcal{CP}$ states and their production and decay exhibits $\mathcal{CP}$ violation. The angular distributions of the Higgsstrahlung cross section depends upon whether the Φ is $\mathcal{CP}$-even, $\mathcal{CP}$-odd, or a mixture [4, 10, 11, 12, 13]. Also the angular distribution of the fermions in the $Z \to f\bar{f}$ from $Z\Phi$ production reflects the $\mathcal{CP}$ nature of the state Φ [4, 10, 12, 14]. An analysis of the angular distributions of the final state fermions in the Higgsstrahlung process with the formalism of optimal variables has been performed in ref.[15]. A fit to double-differential angular distribution in the production and decay angles results in a clean separation between a scalar and pseudoscalar states assuming that the $Z\Phi$ cross section is independent of the $\mathcal{CP}$ nature of the ϕ [16]. Recently, the prospects for the meaurement of the pseudoscalar admixture in the $h\tau\tau$ coupling to a SM Higgs boson was presented [17].

In this paper, we present an alternative method that simultaneouly uses the distributions of the production and decay angles to distinguish the SM-like Higgs boson from a $\mathcal{CP}$-odd 0^{-+} state A, or a $\mathcal{CP}$-violating mixture Φ. We perform an analyisis of Monte Carlo events that takes into account the signals and background, as well as a simulation of a TESLA detector response. In the next section we shall present the theoretical ansatz considered and the details of the Monte Carlo simulation used to generate the $e^+e^- \to Z\Phi$ samples. We then describe the proposed method, detector simulations and the imposed cuts for the event selection. Finally we present the fit techniques and the results obtained from Monte Carlo studies.

$e^+e^- \to Z\Phi$ **SAMPLES**

Events of the signal $e^+e^- \to ZH$ were generated using the PYTHIA program [18]. The cross section for the Higgsstrahlung process is given by:

$$\sigma(e^+e^- \to ZH) = \frac{G_F^2 M_Z^4}{96\pi s}(a_e^2 + v_e^2)\beta\frac{\beta^2 + 12M_Z^2/s}{(1 - M_Z^2/s)^2} \quad (1)$$

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[1] The new world average of the expected Higgs mass of 117 GeV [2] is yet accesible in the current run of the Tevatron

with $\beta = \sqrt{[s - (M_H + M_Z)^2][s - (M_H - M_Z)^2]}/s$.

The effects of initial state bremsstrahlung were included in the PYTHIA generation.

For the production of a Higgs boson Φ with arbitrary $\mathcal{CP}$ properties, $e^+e^- \to Z\Phi$, the amplitude for the process (which is not included in PYTHIA) can be described by adding a ZZA coupling with strength η to the SM matrix element as in ref.[1]. The matrix element of the process containing both the $\mathcal{CP}$ even amplitude, $\mathcal{M}_{ZH}$, and a $\mathcal{CP}$ odd amplitude, $\mathcal{M}_{ZA}$, is given by:

$$\mathcal{M}_{Z\Phi} = \mathcal{M}_{ZH} + i\eta \cdot \mathcal{M}_{ZA} \qquad (2)$$

where η is a dimensionless factor. The total cross section depends of the value of η as follows [15]:

$$\sigma(\eta, s) = \frac{G_F^2 M_Z^6 \beta}{16\pi} \frac{1}{D_Z(s)}(v_e^2 + a_e^2)(2 + \frac{s\beta^2}{6M_Z^2} + \eta^2 \frac{s^2\beta^2}{M_Z^4}) \qquad (3)$$

where

$$D_Z(s) = (s - M_Z^2)^2 + M_Z^2\Gamma_Z^2 \qquad (4)$$

and M_Z, Γ_Z denote the Z boson mass and width, G_F is the Fermi constant, and v_e, a_e are the usual vector and axial-vector coupling constants of the e to the Z boson. In the SM η is zero. In the MSSM a ZZA is forbidden at Born level, but is induced via higher-order loop effects [9]. In general in extensions of the Higgs sector, η need not to be loop suppressed, and may be arbitrarily large. Hence, it is useful to allow for η to be a free parameter in the data analysis.

As it was mentioned in the Introduction, the quantum numbers J^{PC} of the Higgs bosons can be determined at future e^+e^- linear colliders in a model independent way by analysing the angular dependence of the Higgsstrahlung process. The most sensitive kinematic variable to distinguish the different contributions to Higgs boson production is θ, the polar angle of the Z boson w.r.t. the beam axis in the laboratory frame. The sensitivity can be increased by including the angular distributions of the decay to fermions, $Z \to f\bar{f}$ in the boson rest frame (See Fig. 1). Here the z-axis is chosen along the direction of the Z-boson momemtum. The decay amplitude is then a function of the angle between the Z momentum and f, θ^*, and the angle between the Z production plane and the Z decay plane, ϕ^*.

To obtain the angular distributions corresponding to the non-SM Higgs Φ with arbitrary $\mathcal{CP}$ properties in the process $e^+e^- \to Z\Phi$, we have used a "re-weighting" method. This procedure allows one to obtain the distributions for arbitrary values of η by weighting the distributions for $\eta = 0$ according to the differential cross section in θ, θ^*, ϕ^*. The weight factor is given by the
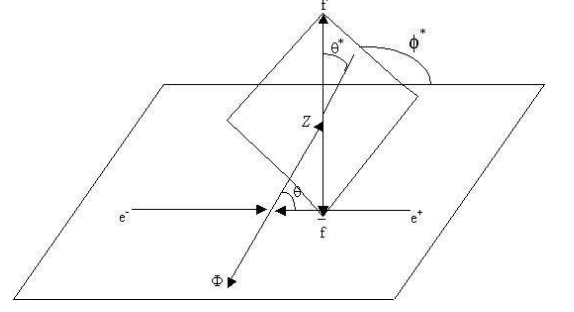

FIG. 1: Definition of the production and decay angles of the process $e^+e^- \to Z\Phi[Z \to f\bar{f}]$.

following ratio:

$$W(\cos\theta, \cos\theta^*, \cos\phi^*) = \frac{|\mathcal{M}_{Z\Phi}(\eta)|^2}{|\mathcal{M}_{ZH}|^2} \qquad (5)$$

The squared amplitude $|\mathcal{M}_{Z\Phi}(\eta)|^2$ has three contributions:

$$|\mathcal{M}_{Z\Phi}(\eta)|^2 = |\mathcal{M}_{ZH}|^2 + \eta \cdot 2\Im m(\mathcal{M}_{ZH}^* \mathcal{M}_{ZA}) + \eta^2 |\mathcal{M}_{ZA}|^2 \qquad (6)$$

The first term reproduces the SM-like cross section. The interference term between the $\mathcal{CP}$ even and $\mathcal{CP}$ odd amplitudes, linear in η, generates a forward-backward asymmetry, that is a hallmark of $\mathcal{CP}$ violation. The third term correspond to the pseudoescalar Higgs cross section. Of course, $\eta = 0$ brings us back to the scalar SM Higgs production. The explicit expression for the squared amplitude of the bremsstrahlung process in terms of θ, θ^*, ϕ^* is taken from ref. [10]. In our procedure, the weight is re-scaled to be lower than 1, for a better treatment of errors. To check the reliability of the method we compared the obtained distributions using Monte Carlo with the analytical expressions. Figure 2 shows the obtained production angular distribution for the process $e^+e^- \to ZA$ using the procedure described above along with the analytical form. The distribution is in very good agreement with the theoretical expectation proving the validity of the "reweighting" procedure.

DESCRIPTION OF THE METHOD AND MONTE CARLO STUDIES

We consider the production of Higgs events at the TESLA operating at a center-of-mass energy of 350 GeV, assuming an integrated luminosity of 500 fb^{-1}. At this energy the main production process for the Higgs boson in the SM is the Higgsstrahlung process, $e^+e^- \to ZH$ [19]. The corresponding expected number of events for this process is 6.6×10^4.

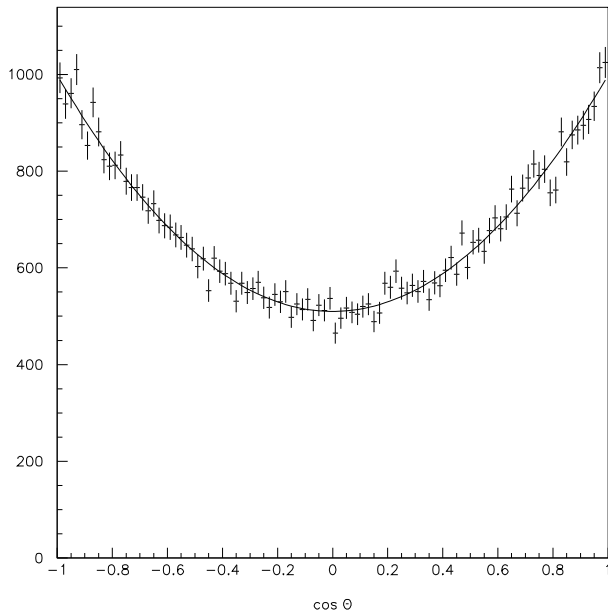We have choosen for the present study the process $e^+e^- \to ZH \to \mu^+\mu^- H$ with a Higgs boson mass of

FIG. 2: Angular distribution of the process $e^+e^- \to ZA$ obtained by the "re-weighting" method (See text) for an integrated luminosity of 500 fb^{-1} and a center of mass energy of $\sqrt{s} = 350$ GeV, assuming a Higgs mass of 120 GeV. The line indicates the exact theoretical dependence.

120 GeV. This decay channel exhibits a clean signature in the detector and the selection efficiencies are expected to be independent of the decay mode of the Higgs boson. We allow the produced Higgs to be either scalar or a mixture state Φ including an interference term.

All the Monte Carlo samples have been generated with the PYTHIA program as described in the previous section. These events are then passed through the simulation package SIMDET [20], a parametric Monte Carlo program for a TESLA detector [21] which follows the proposal presented in the TESLA Conceptual Design Report [1]. For the Higgs boson all decay modes are simulated as expected in the SM. The following background processes are considered in the analysis: $e^+e^- \to e^+e^-f^+f^-$, $e^+e^- \to f^+f^-(\gamma)$, $e^+e^- \to W^+W^-$ and $e^+e^- \to ZZ$. Both signal and background events are processed by the detector simulation package.

For the event selection we follow ref. [15]. At least one muon and anti-muon are identified, with energy larger than 15 GeV. The mass of the di-muon system is required to be consistent with the Z boson hypothesis within 5 GeV. The recoil mass of the di-muon system $M_{rec}^2 = (\sqrt{s} - (E_{\mu^+} + E_{\mu^-}))^2 - (\vec{P}_{\mu^+} + \vec{P}_{\mu^-})^2$ has to be consistent with the H boson hypothesis within 5 GeV. This variable will yield a peak for the signal of the Higgs boson mass, independently of the Higgs boson decay mode. To remove a significant part of the remaining background, the absolute z-component of the di-muon system is required to be smaller than 120 GeV.

The momemtum of the selected muons are used to cal-

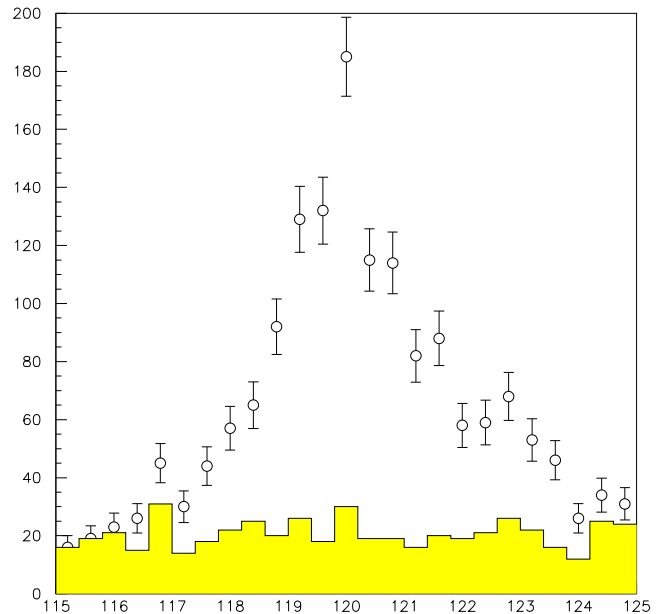FIG. 3: Recoil mass spectra off the Z in $e^+e^- \to ZH \to \mu^+\mu^- X$. Shadowed area represents the expected background events.

culate the cosines of the production and decay angles for futher use in the method to determine the J^{PC} properties of the Higgs boson. It has been noted in [1] that having excellent momemtum and energy resolution will allow the Z to be well reconstructed. The recoil mass against the Z, can then be used to detect the Higgs boson and to study its properties. Figure 3 shows the recoil mass distribution for the $m_H = 120$ GeV signal, obtained from the selected events in the sample of $e^+e^- \to ZH \to \mu^+\mu^- X$. The Higgs boson signal appears on top of a small background. In Figure 4 the corresponding $\cos\theta$ distribution is shown. The expected background is also presented. The combination of the cut on the z-component of the di-muon system, and the decreasing muon identification performance results in an efficiency for $\cos\theta > 0.9$ close to zero.

The kinematics of the $e^+e^- \to Z\Phi$ $[Z \to f\overline{f}]$ process is described by the production and decay angles θ, θ^*, ϕ^*. The method we propose consists in generating 3-dimensional distributions in $\cos\theta$, $\cos\theta^*$ and $\cos\phi^*$ using the Monte Carlo events generated as described above for each contribution in equation (3). We write then the likelihood:

$$\mathcal{L} = \prod_{(\cos\theta)_i, (\cos\theta^*)_j, (\cos\phi^*)_k} \frac{\mu_{ijk}^{N_{data}(i,j,k)} e^{-\mu_{ijk}}}{N_{data}(i,j,k)!} \quad (7)$$

where $N_{data}(i,j,k)$ is the number of events of the hypothetical data sample and μ_{ijk} is the expected number in the ijk-th bin. μ_{ijk} is calculated assuming a linear combination of the number of events of three Monte Carlo
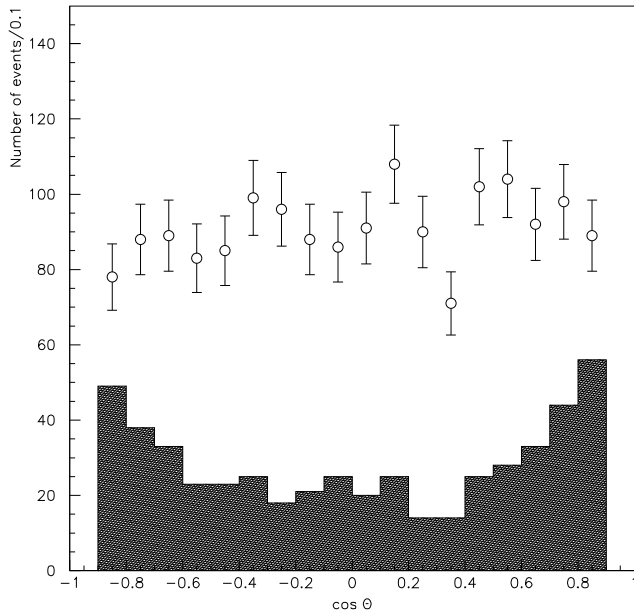
FIG. 4: Angular distribution, $\cos\theta$, of the selected events in $e^+e^- \to ZH \to \mu^+\mu^- X$. The shaded histogram correspond to the expected background.

samples, corresponding to the production of scalar Higgs (MC_ZH), pseudoscalar (MC_ZA) Higgs and events for the interference term (MC_IN):

$$\mu_{ijk} = \mathcal{N}.(\alpha.MC\_ZH_{ijk} + \beta.MC\_IN_{ijk} + \gamma.MC\_ZA_{ijk}) \quad (8)$$

where $\mathcal{N}$ is the overall normalization factor between numbers of data and Monte Carlo events which can be fixed ($\mathcal{N} = 0.1$ in our case) or left free as a further check of the fit. The likelihood is then maximized with respect to α, β and γ. The absolute value of β indicates the contribution of interference term in the sample and α and γ indicate the fraction of scalar and pseudoscalar components respectively. A significant deviation of β from zero would imply the existence of $\mathcal{CP}$ violation, independent of the specific model. For a scalar Higgs sample ($\eta = 0$), the result of the fit is expected to be $\alpha = 1$ and $\beta = \gamma = 0$.

We have performed Monte Carlo studies with several hypothetical data samples with non-standard values of η. A maximum likelihood fit for the best linear combination of MC_ZH, MC_ZA and MC_IN to match the hypothetical data sample gave statistical errors of 0.04, 0.02 and 0.04 for α, β and γ, respectively. The results of these studies using different values of η are given in table I.

The value of α gives the fraction of the scalar $J^{PC} = 0^{++}$ component of the Higgs boson, while γ gives the contribution of the pseudoscalar Higgs component and increases quickly with η as expected. It can be seen from our results that the Monte Carlo study using a sample of pure scalar SM-like Higgs gives a consistent answer. This indicates the high sensitivity of the method to distinguish a purely $\mathcal{CP}$-even state from a pseudoscalar $\mathcal{CP}$-odd

TABLE I: Obtained values of the parameters for differents "data" samples

η	α	β	γ
-0.4	0.002 ± 0.03	-0.05 ± 0.02	0.98 ± 0.04
-0.25	0.08 ± 0.04	-0.06 ± 0.02	0.92 ± 0.04
-0.1	0.43 ± 0.04	-0.09 ± 0.02	0.57 ± 0.04
-0.05	0.69 ± 0.04	-0.06 ± 0.02	0.31 ± 0.04
0	0.97 ± 0.05	0.003 ± 0.02	0.03 ± 0.04
0.05	0.70 ± 0.05	0.05 ± 0.02	0.29 ± 0.04
0.1	0.40 ± 0.04	0.04 ± 0.02	0.59 ± 0.04
0.25	0.08 ± 0.04	0.04 ± 0.02	0.92 ± 0.04
0.4	0.002 ± 0.03	0.01 ± 0.02	0.98 ± 0.04

state. Secondly, the method also allows one to determine whether the observed Higgs boson is a $\mathcal{CP}$ mixture and, if so, measure the odd and even component. It is evident that the statistical uncertainties prevent us to a large extent from measuring the interference term. It should be noted that for $Z \to \mu^+\mu^-$, as well as for $Z \to e^+e^-$, the interference term is suppressed by the smallness of v_f independently of the size of η. However, the simultaneous existence of fractions α and γ would indicate $\mathcal{CP}$ violation for the $ZZ\phi$ coupling. The method proposed here gives sensible results in the case that there is any significant $\mathcal{CP}$-even component in the ϕ Higgs boson or if ϕ is almost purely $\mathcal{CP}$-odd. The statistical significance can certainly be increased including the $e^+e^- \to e^+e^- X$ channel.

SUMMARY

We have proposed a novel method for the measurement of the parity of the Higgs boson using the angular distributions of the differential cross section of $e^+e^- \to Z\phi$. The statistical power of our method using Monte Carlo generated hypothetical data samples is shown in Table I. The results indicate that, for an integrated luminosity of $500 fb^{-1}$, at 350 GeV centre-of-mass energy, TESLA will be able to unambiguosly determine whether a Higgs boson is a state 0^{++} ($\mathcal{CP}$-even, scalar) or has a contribution of the 0^{-+} ($\mathcal{CP}$-odd, pseudoscalar) state, like in general extensions of Higgs model. We also estimate the statistical uncertainties for the measurement of the $\mathcal{CP}$ violating interference term. We hope that this technique will allow confirmation of the expected J^{PC} assignment of a Higgs boson candidate.

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[1] F. Richard, J.R. Schnieder, D. Trines, A. Wagner. *TESLA Technical Design Report* (2001).

[2] D0 Collaboration, V.M. Abazov et al. arXiv:hep-ex/0406031

[3] E. Accomando, *et al.* ECFA/DESY LC, Phys. Rep.**299** 1 (1998).

[4] V. Barger, K. Cheung, A Djoundi, B.A. Kniehl, P.M. Zerwas. Phys. Rev. D, **49** 1 (1994).

[5] B.A. Kniehl, Phys. Rep. **240**, 211 (1994)

[6] J.F. Gunion, H.E. Haber and R.V.Kooten, in "Higgs Physics at the LC" edited by K. Fujii, D.Miller and A.Soni (World Scientific) arXiv:hep-ph/0301023.

[7] M.T.Dova, P. Garcia-Abia and W. Lohmann. LC-PHSM-2001-054 (http://www.desy.de/~lcnotes/) and [arXiv:hep-ph/0302113]

[8] P. W. Higgs, Phys. Rev. Lett. **12**, 132 (1964).

[9] J. Gunion, H. Haber, G. Kane and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Reading, 1990)

[10] A. Djouadi, B. Kniehl. DESY 93-123 p. 51.

[11] D.J. Miller, S.Y. Choi, B. Eberle, M.M.Muhlleitner, P.M. Zerwas. Phys. Lett. **505**, 149 (2001).

[12] K. Hagiwara and M.L.Stong, Z.Phys C62,99 (1994). K.Hagiwara, S.Ishihara, J.Kamoshita and B.A. Kniehl, Eur. Phys. J.**C14**, 457 (2000).

[13] T.Han and J.Jiang, Phys. Rev. **D63**, 096007 (2001).

[14] B.A. Kniehl. Int.J.Mod.Phys. **A17** (2002) 1457-1476. [arViv:hep-ph/0112023].

[15] M. Schumacher, LC-PHSM-2001-003.

[16] V.D. Derwent et al.,*Linear Collider Physics* FERMILAB-FN-701 [arXiv:hep-ex/0107044]

[17] K. Desch, A. Imhof, Z. Was and M.Worek, Phys. Lett. B579 157 (2004).

[18] T. Sjöstrand, Comp. Phys. Comm. **135**, 238 (2001).

[19] J. Ellis, M.K. Gaillard, D.V. Nanopoulos. Nucl. Phys. **B106**, 292 (1976); B.L. Ioffe, V.A. Khoze. Sov. J. Nucl. Phys. **9**, 50 (1978); B.W. Lee, C. Quigg, H.B. Thacker. Phys. Rev. **16**,1519 (1977).

[20] M.Pohl and H.J.Schreiber, report DESY 02-061, [arXiv:hep-ex/0206009] and LC-DET-2002-005.

[21] T. Behnke, S. Bertolucci, R.D. Heuer and R. Sttels, TESLA Technical Design Report Part IV: A detector for TESLA, DESY-01-011